B.Sc VIth Sem

Subject : Abstract Algebra (Paper I)

UNIT I

Q.1 : A non-empty set A is termed as an algebraic structure

a) with respect to binary operation *

b) with respect to binary operation ?

c) with respect to binary operation +

d) with respect to binary operation –

Ans : a

Q.2 : A group (G,*) is said to be abelian if

a) (x + y) = (y + x)

b) (x * y) = (y * x)

c) (x + y) = x

d) (y * x) = (x + y)

Ans : b

Q.3 : Matrix multiplication is a/an

a) Commutative

b) Associative

c) Additive

d) Distributive

Ans : b

Q.4 : How many properties can be held by a group?

a) 2

b)3

c) 5

d) 4

Ans : c

Q.5 : How many binary operation group is defined?

a) 2

b) 4

c) 1

d) None of these

Ans : a

Q.6 : If under the product in G, H itself forms a group is called

a) Normal subgroup

b) Subgroup

c) Group

d) All of the above

Ans : b

Q.7 : If H is a subgroup of a finite group G, then o(H)|o(G)

a) Euler's theorem

b) Fermat's theorem

c) Lagrange's theorem

d) None of these

Ans : c

Q.8 : The statement of Fermat's theorem is

a) If G is a finite group and a $\in G, then\ o(a)|o(G)$

b) If n is a positive integer and a is relatively prime to n, then $a^{\emptyset(n)} \equiv 1(mod\ n)$

c) If P is a prime number and a is any integer, then $a^p \equiv a(mod\ p)$

d) If G is a finite group and a $\in G, then\ a^{o(G)} = e$

Ans : c

Q.9 : The subgroup N of G is a normal subgroup of G if and only if

a) $gng^{-1} = N, \forall g \in G$

b)$gN = Ng, \forall\ g \in G$

c) Both (a) and (b)

d) None of these

Ans : c

Q.10 : Fundamental theorem on homomorphism is

a) If Ø is a homomorphism of G into G' with kernel K, then K is normal subgroup of G

b) A homomorphism Ø of G into G' with kernel K is an isomorphism of G into G' iff KerØ $= \{e\}$, where e is identity of G'

c) If Ø is a homomorphism of G into G' with kernel K, then G/K ≈ G'

d) All of the above

Ans : c

Q.11 : A cyclic group is always

a) abelian group

b) monoid

c) semigroup

d) subgroup

Ans : a

Q.12 : The set $N(a) = \{x \in G | xa = ax\}$ is

a) Conjugate class

b) Normalizer

c) Conjugacy relation

d) None of these

Ans : b

Q.13 : Which sentence is true?

a) Set of all matrices forms a group under multiplication

b) Set of all rational negative numbers forms a group under multiplication

c) Set of all non-singular matrices forms a group under multiplication

d) Both (b) and (c)

Ans : c

Q.14 : The set of all real numbers under the usual multiplication operation is not a group since

a) multiplication is not a binary operation

b) Multiplication is not associative

c) Identity element does not exist

d) Zero has no inverse

Ans : d

Q.15 : If G is a group, p is a prime positive integer and p|o(G), then G has an element of order p

a) Euler's theorem

b) Cauchy theorem

c) Lagrange's theorem

d) None of these

UNIT II

Q.1 : A nonempty subset S of a vector space V is a subspace of V iff

a) If $u, v \in S, then\ u + v \in S\ and\ u \in S\ and\ \alpha\ a\ scalar, then\ \alpha u \in S$

b) $\alpha u + \beta v \in S, \forall u, v \in S\ and\ all\ scalars\ \alpha, \beta$

c) Both (a) and (b)

d) None of these

Ans : c

Q.2 : If U and W be two subspaces of vector space V then $U \cup W$ is a subspace of V iff

a) $U \subset W$

b) $W \subset U$

c) Either (a) or (b)

d) None of these

Ans : c

Q.3 : The vectors (1, 0, 1), (1, 1, 0) and (1, 1,-1) are

a) LI

b) LD

c) Neither (a) nor (b)

d) Both (a) and (b)

Ans : a

Q.4 : Let V be any vector space. Then

a) The set {v} is LD iff v = 0

b) The set $\{v_1, v_2\}$ is LD iff v_1 and v_2 are collinear

c) The set $\{v_1, v_2, v_3\}$ is LD iff v_1, v_2 and v_3 are coplanar

d) All of the above

Ans : d

Q.5 : The vectors (1, 1, 1), (1, -1, 1) and (3, -1,3) are

a) LI

b) LD

c) Neither (a) nor (b)

d) Both (a) and (b)

Ans : b

Q.6 : The Vectors (a, b) and (c, d) are L.D. iff

a) ad $\neq$ bc

b) ad = bc

c) Both (a) and (b)

d) None of these

Ans : b

Q.7 : $\{(x_1, x_2, x_3, x_4),(y_1, y_2, y_3, y_4),(z_1, z_2, z_3, z_4)\}$ is L.D. iff

a) $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} \neq 0$

b) $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = 0$

c) Both (a) and (b)

d) None of these

Ans : b

Q.8 : A subset B of a vector space V is said to be basis for V if

a) B is L.I.

b) B generates V

c) Neither (a) nor (b)

d) Both (a) and (b)

Ans : d

Q.9 : In a vector space V if S = {v_1, v_2, ….., v_n} generates V and if {w_1, w_2,….., w_m} is L.I. then

a) $m < n$

b) $m > n$

c) $m \leq n$

d) $m \geq n$

Ans : c

Q.10 : If V has a basis of n elements, then every set of p vectors with $p > n$, is

a) LI

b) LD

c) Neither (a) nor (b)

d) Both (a) and (b)

Ans : b

Q.11 : If a vector space V is n-dimensional then

a) There exist n L.I. vectors in V

b) Every set of (n+1) vectors in V is L.D.

c) Both (a) and (b)

d) None of these

Ans : c

Q.12 : If V has a basis of n elements, then every other basis for V has

a) n+1 elements

b) n elements

c) no elements

d) no elements

Ans : b

Q.13 : If U and W are two subspaces of a finite dimensional vector space V, then

a)$\dim(U+W) = dimU + dimW + \dim(U \cap W)$

b)$\dim(U+W) = dimU + dimW - \dim(U \cap W)$

c)$\dim(U+W) = dimU - dimW + \dim(U \cap W)$

d)$\dim(U+W) = dimU - dimW - \dim(U \cap W)$

Ans : b

Q.14 : If U and W are two subspaces of a finite dimensional vector space V such that

a) $U \cup W = \{0\}$

b) $U \cap W = \{0\}$

c) Neither (a) nor (b)

d) Both (a) and (b)

Ans : b

Q.15 : Let U be a subspace of a finite-dimensional vector space V then

a)$\dim V < \dim U$

b)$\dim U > \dim V$

c)$\dim U \leq \dim V$

d)$\dim U \geq \dim V$

Ans : c

UNIT III

Q.1 : Let U and V be vector spaces over the same field F. A mapping T : U → V is Linear if and only if

a) $\mathrm{T}(\alpha u_1 + \beta u_2) = \alpha T(u_1) + \beta T(u_2), \forall\, u_1, u_2 \in U \; and \; \alpha, \beta \in F$

a) $\mathrm{T}(\alpha u_1 + u_2) = \alpha T(u_1) + T(u_2), \forall\, u_1, u_2 \in U \; and \; \alpha, \in F$

c) Both (a) and (b)

d) None of these

Ans : c

Q.2 : Let T : $V_3 \rightarrow V_3$ defined by T(x_1, x_2, x_3) = (x_1, x_2, 0) is a

a) Non-linear transformation

b) Linear transformation

c) Only (b)

d) None of these

Ans : b

Q.3 : : Let U and V be vector spaces over the same field F. A mapping T : U → V is Linear map then

a) T(0) = 0

b) T(-u) = -T(u)

c) Both (a) and (b)

d) Neither (a) nor (b)

Ans : c

Q.4 : Let T : U → V be a Linear map. Then

a) R(T) is a subspace of V

b) N(T) is a subspace of V

c) T is one-one iff N(T) is the zero subspace of U

d) All of the above

Ans : d

Q.5 : If U is finite-dimensional then

a)$\dim R(T) < \dim U$

b)$\dim R(T) > \dim U$

c)$\dim R(T) \leq \dim U$

d)$\dim R(T) \geq \dim U$

Ans : c

Q.6 : Let T : U → V be a Linear map and U be a finite-dimensional vector space then

a) dim R(T) + dim U = dim N(T)

b) dim R(T) + dim N(T) = dim U

c) dim R(T) – dim N(T) = dim U

d) None of these

Ans : b

Q.7 : If U and V are finite-dimensional vector spaces of same dimension, then a linear map $T : U \rightarrow V$ is one-one iff

a) T is not onto

b) T is onto

c) Neither (a) nor (b)

d) Both (a) and (b)

Ans : b

Q.8 : Let $T : U \rightarrow V$ and $S : V \rightarrow W$ be two Linear map. Then

a) If ST is onto, then S is onto

b) If ST is one-one, then T is one-one

c) If ST is singular, then T is one-one and S is onto

d) Both (a) and (b)

Ans : d

Q.9 : Let $T : U \rightarrow V$ and $S : V \rightarrow W$ be two Linear map. If U, V, W are of the same finite dimension and ST is non-singular then

a) S and T are singular

b) S and T are non-singular

c) S is singular and T is non-singular

d) T is singular and S is non-singular

Ans : b

Q.10 : Let $T : V_3 \rightarrow V_3$ be a linear map defined by $T(x_1, x_2, x_3) = (x_1, x_2, 0)$. Then Kernel of T is

a) [(0, 0, 2)]

b) [(1, 0, 0)]

c) [(0, 0,1)]

d) None of these

Ans : c

Q.11 : Let T : U → V be a linear map then

a) If T is one-one and $u_1, u_2, \ldots, u_n$ are LI vectors of U, then $T(u_1), T(u_2), \ldots, T(u_n)$ are LI vectors in V

b) If $v_1, v_2, \ldots, v_n$ are LI vectors of R(T) and $u_1, u_2, \ldots, u_n$ are vectors in U such that $T(u_i) = v_i$, for i = 1, 2, …, n then $\{u_1, u_2, \ldots, u_n\}$ is LI

c) Both (a) and (b)

d) only (b)

Ans : c

Q.12 : Let $T : V_1 \rightarrow V_3$ defined by T(x) = (x, 2x, 3x) is

a) Linear Transformation

b) not- linear Transformation

c) Neither (a) nor (b)

d) None of these

Ans : a

Q.13 : Let T : U → V be a non-singular linear map. Then $T^{-1}: V \rightarrow U$ is

a) Linear

b) one-one

c) onto

d) All of the above

Ans : d

Q.14 : Let T_1, T_2 be a linear maps from U to V. Let S_1, S_2 be a linear maps from V to W then

a) $S_1 (T_1+T_2) = S_1T_1 + S_1T_2$

b) $(S_1 + S_2)T_1 = S_1T_1 + S_2T$

c) $(\alpha S_1)T_1 = \alpha(S_1T_1)$, where α is a scalar

d) All of the above

Ans : d

Q.15 : : Let U and V be vector spaces over the same field F. A mapping T : U → V is Linear if and only if $T(\alpha u_1 + \beta u_2) = \alpha T(u_1) + \beta T(u_2), \forall\, u_1, u_2 \in U \; and \; \alpha, \beta \in F$ is called

a) Linear transformation

b) Non-linear transformation

c) Neither (a) nor (b)

d) Both (a) and (b)

Ans : a

UNIT IV

Q.1 : The dimension of kernel of matrix A is called

a) Rank

b) Kernel

c) Range

d) Nullity

Ans : d

Q.2 : The dimension of the range of matrix A is called

a) Rank

b) Kernel

c) Range

d) Nullity

Ans : a

Q.3 : A square matrix is non-singular iff

a) its column and row vectors are L.D.

b)its column and row vectors are L.I .

c) its column and row vectors are not L.D.

d) its column and row vectors are not L.D.

Ans : b

Q.4 : An $n \times n$ matrix A is invertible iff the corresponding linear transformation T is

a) Singular

b) Non-singular

c) Singular as well as non-singular

d) Neither singular nor non-singular

Ans : b

Q.5 : Let A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is non-singular. Find its inverse

a)$A^{-1} = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

b) $A^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

c) $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

d) None of these

Ans : c

Q.6 : Which of the following is true?

a) $(u + v, w) = (u, w) + (v, w)$

b) $(u, \alpha u) = \bar{\alpha}(u, v)$

c)$(0, u) = (u, o) = 0$

d) All of the above

Ans : d

Q.7 : Inner product space is defined on

a) real vector space

b) complex vector space

c) both (a) and (b)

d) none of these

Ans : c

Q.8 : If $u, v \in V \ then \ |u, v| \leq ||u|| \ ||v||$ then

a) Norm

b) Cauchy-Schwarz inequality

c) Both (a) and (b)

d) None of these

Ans : b

Q.9 : Gram-Schmidt orthogonal process is

a) The set of vectors $\{v_i\}$ in V is an orthogonal set if $(v_i, v_j) = 0$ for $i \neq j$

b) Any orthogonal set of nonzero vectors in an inner product space is linearly independent.

c) Every finite dimensional inner product space has an orthogonal basis.

d) None of these

Ans : c

Q.10 : Let V be an inner product space and $u, v \in V$ then

a) $||u|| \geq 0$

b) $\left||u|\right| = 0 \; iff \; u = 0$

c) $\left||u + v|\right| \leq \left||u|\right| + ||v||$

d) All of the above

Ans : d

Q.11 : A matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is

a) Orthogonal matrix

b) Hermitian matrix

c) Unitary matrix

d) None of these

Ans : c

Q.12 : If H is orthogonal matrix then |H| is

a) 1

b) -1

c) 0

d) Both (a) and (b)

Ans : d

Q.13 : If U is unitary matrix then |U| is

a) 1

b) -1

c) 0

d) None of these

Ans : a

Q.14 : If U is unitary matrix then

a) U^{-1} is not Unitary matrix

b) U^{-1} is Unitary matrix

c) Neither (a) nor (b)

d) None of these

Ans : b

Q.15 : A matrix $\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ is a/an

a) Unitary matrix

b) Nilpotent matrix

c) Orthogonal matrix

d) Symmetric matrix

Ans : c